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Robust Tunable Trajectory Repairing for Autonomous Vehicles Using Bernstein Basis Polynomials and Path-Speed Decoupling

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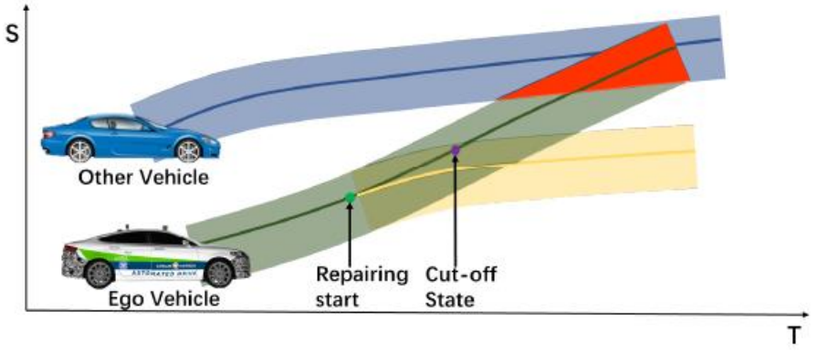
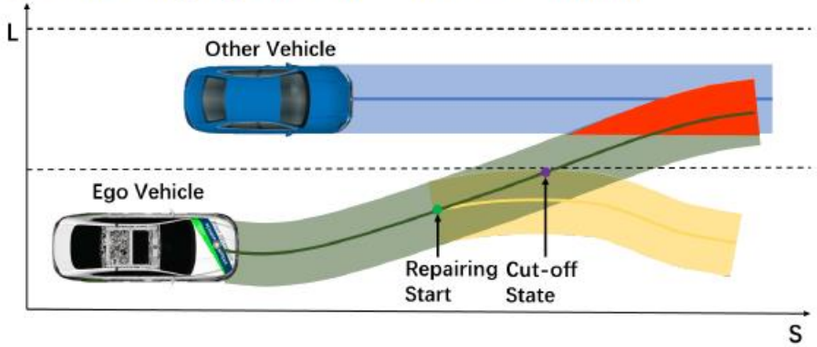
Motivation

- **Trajectory repairing**
  - **Assumption**: **the intention of other vehicles changes, causing a hazardous situation.**
  - Detect the part of an invalid trajectory that can stay unchanged.
  - Repair the remaining part.
  - Pros
    - Not need to replan the whole trajectory
    - More robust against small disturbances

- **Cut-off state**
  - The time that evasive maneuvers must be taken to avoid a collision.



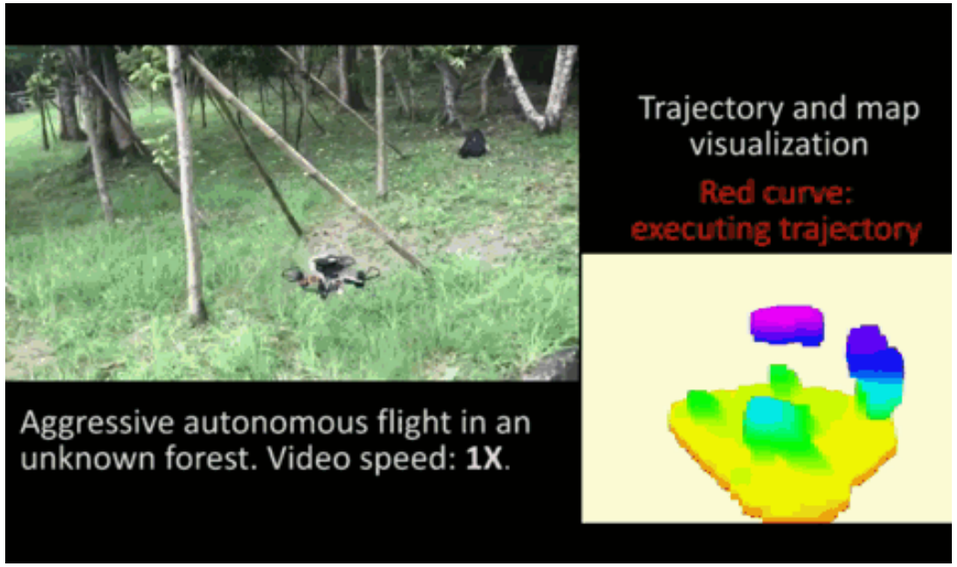
(a) Speed Repairing in S-T domain. The motion of the ego vehicle and the other vehicle in the current ego lane is projected into the S-T domain



(b) Path Repairing in L-S domain. The motion of the ego vehicle and the other vehicle is projected into the L-S domain in curvilinear coordinates.

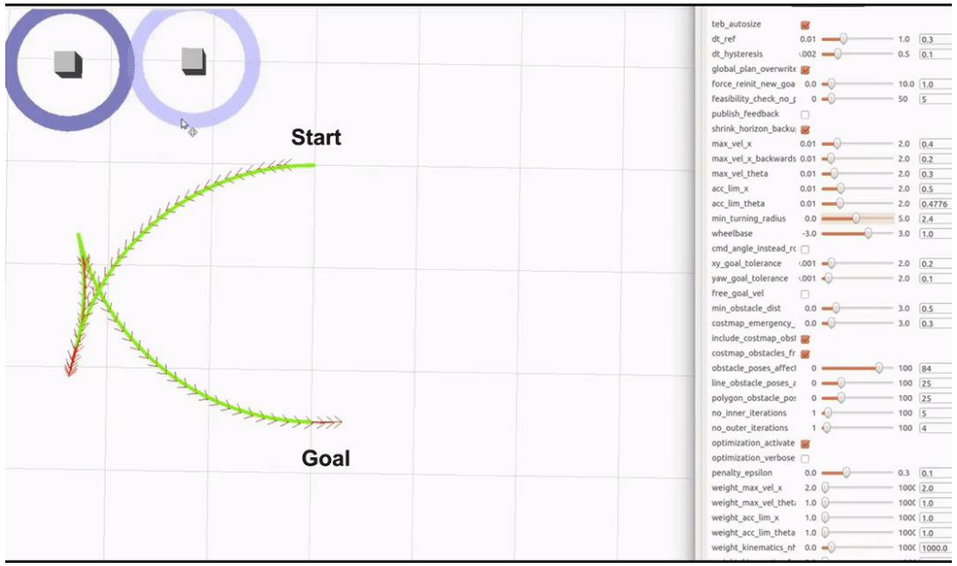
Background

- There are similar concepts for car-like robots, or drones. [1]
- Such as: local planning, trajectory deformation, elastic band..



- However, it is not specifically intended for autonomous driving, and is not connected to safety assurance. When the “repair” happens depends on its discretization.

[2]



[1] B. Zhou, F. Gao, L. Wang, C. Liu and S. Shen, "Robust and Efficient Quadrotor Trajectory Generation for Fast Autonomous Flight," in IEEE Robotics and Automation Letters, vol. 4, no. 4, pp. 3529-3536, Oct. 2019, doi: 10.1109/LRA.2019.2927938.
[2] C. Rösmann, F. Hoffmann and T. Bertram: Integrated online trajectory planning and optimization in distinctive topologies, Robotics and Autonomous Systems, Vol. 88, 2017, pp. 142–153.

Original trajectory repairing [3]

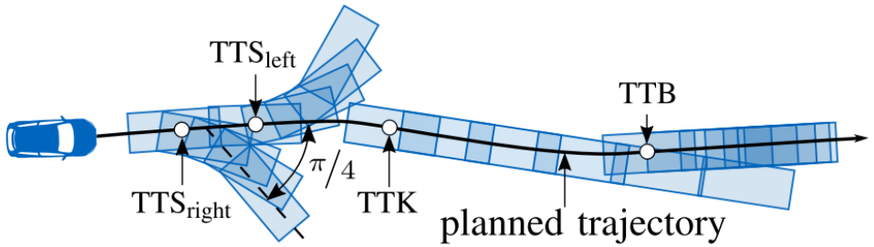
1) Search for Time-To-Brake (TTB), Time-to-Kickdown(TTK), Time-To-Steer (TTS)

2) Time-To-React (TTR) is the maximum among them.

3) Repair starts from TTR

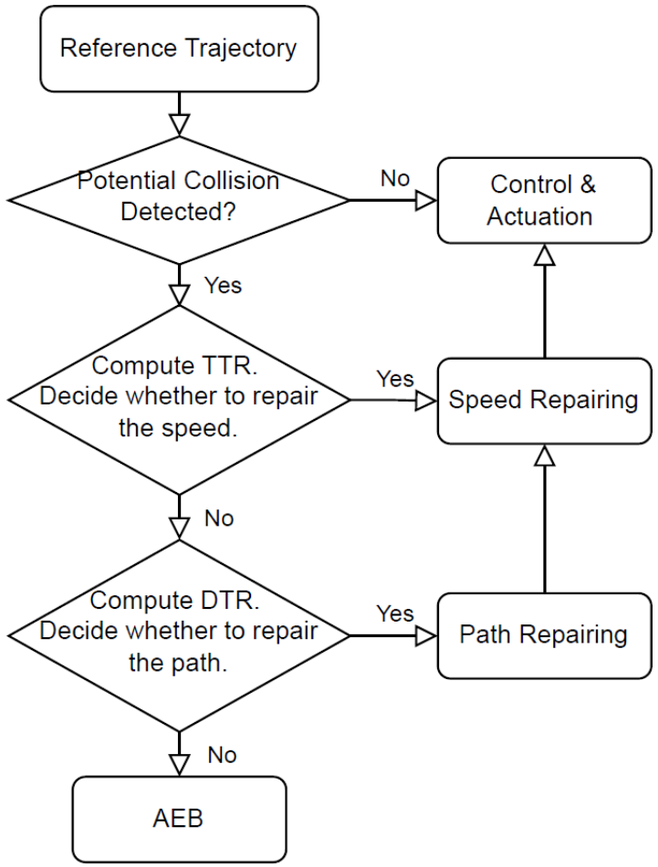
TABLE I: Description of evasive maneuvers.

Metric	Description
TTB	Full braking with maximum deceleration
TTK	Full acceleration until reaching the maximum velocity
TTS	Full steering to the left or right with maximum steering angle until the relative orientation change is equal to $\pm\ \pi/4$

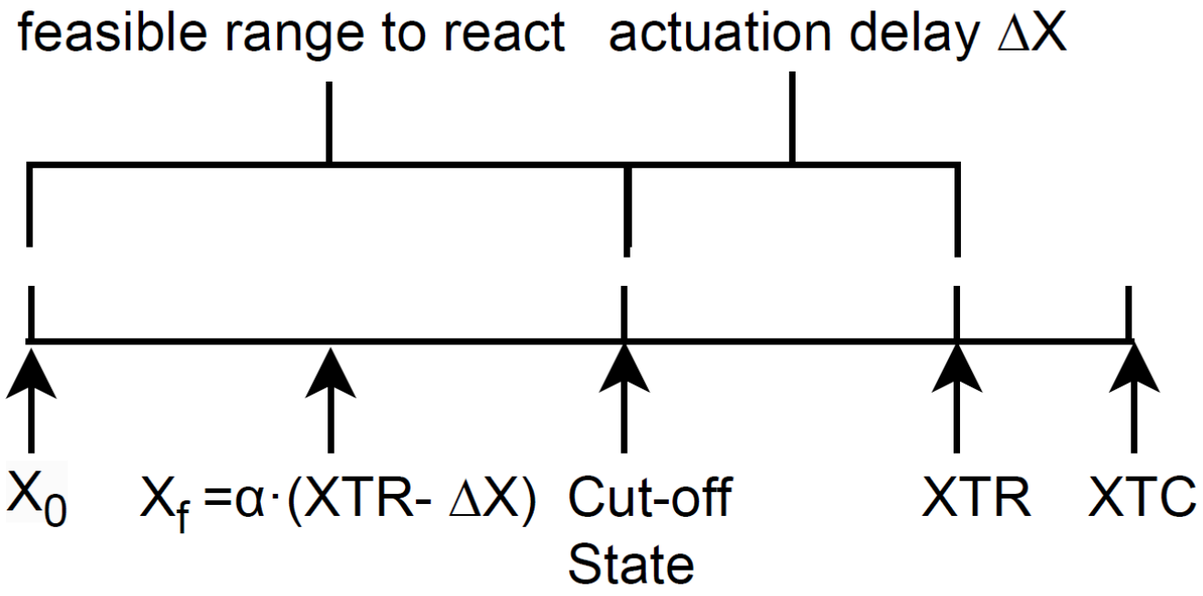


[3] Lin, Yuanfei; Maierhofer, Sebastian; Althoff, Matthias (2021 - 2021): Sampling-Based Trajectory Repairing for Autonomous Vehicles. In : 2021 IEEE International Intelligent Transportation Systems Conference (ITSC). 2021 IEEE International Intelligent Transportation Systems Conference (ITSC).

Our idea: path-speed decoupling



α-robustness for reparing



XTC: Time-to-Collision or Distance-to-Collision

XTR: Time-to-React or Distance-to-React

ΔX: ΔT or ΔS

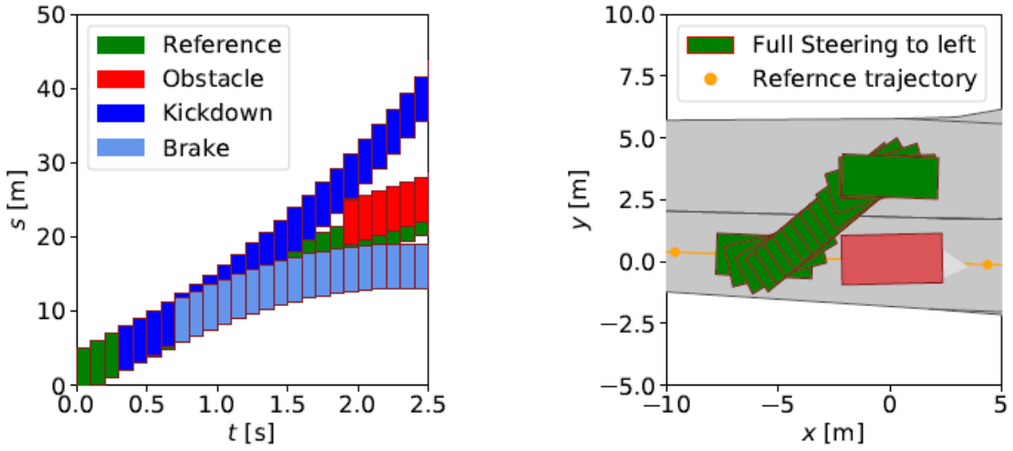
α : a metric in [0, 1]

Hierarchical Search for XTR

Algorithm 1 Hierarchical Search For XTR

Require: P_0: Set of the reference trajectory and predicted trajectories of other vehicles

$M_{speed} \leftarrow setSpeedEvasiveManeuvers(P_0)$
$TTC, DTC \leftarrow detectCollision(P_0)$
if $TTC == 0$ **then**
  $XTR \leftarrow 0$, **return** XTR
else if $TTC == \infty$ **then**
  $XTR \leftarrow \infty$, **return** XTR
else
  for $m \in M_{speed}$ **do**
    $TTM_m \leftarrow searchTTM(m, TTC, P_0)$
  end for
  $XTR \leftarrow max\{TTM_m | m \in M_{speed}\}$
  $m_{speed} \leftarrow argmax\{TTM_m | m \in M_{speed}\}$
  if isManeuverSpeedProper(XTR, m_{speed}) **then**
    return XTR
  end if
end if
$M_{path} \leftarrow setPathEvasiveManeuvers(P_0)$
for $m \in M_{path}$ **do**
  $DTM_m \leftarrow searchDTM(m, DTC, P_0)$
end for
$XTR \leftarrow max\{DTM_m | m \in M_{path}\}$
return XTR



(a) Search for TTR. (b) Search for DTR.

Fig. 6: Exemplary results of binary search in S-T domain and X-Y domain.

Optimization Problem for both path and speed optimization

Bézier Curve

$$B(x) = c^0 b_n^0(x) + c^1 b_n^1(x) + \cdots + c^n b_n^n(x) = \sum_{i=0}^{n} c^i b_n^i(x)$$

Bézier Piecewise trajectory

$$f(x) = \begin{cases} h_0 B_0(\frac{x-X_0}{h_0}), x \in [X_0, X_1) \\ h_1 B_1(\frac{x-X_1}{h_1}), x \in [X_1, X_2) \\ \dots \\ h_{m-1} B_{m-1}(\frac{x-X_{m-1}}{h_{m-1}}), x \in [X_{m-1}, X_m] \end{cases}$$

Quadratic Programming!

$$\min_{\mathbf{c}} \quad \mathbf{c}^T Q_c \mathbf{c} + \mathbf{p_c}^T \mathbf{c} + const$$

$$s.t. \quad A_{eq}\mathbf{c} = \mathbf{b_{eq}}$$

$$A_{ie}\mathbf{c} \leq \mathbf{b_{ie}}$$

Optimization Problem

$$\boldsymbol{f(x) = s(t) \text{ or } l(s)}$$

Objective function

$$J = w_1 \int_0^X (f(x) - r(x))^2 dx + w_2 \int_0^X (f'(x) - r')^2 dx + w_3 \int_0^X f''(x)^2 dx + w_4 \int_0^X f'''(x)^2 dx + w_5 (f(X) - r(X))^2$$

1) Boundary Constraints:

$$(h_0)^{1-l} c_0^{0,l} = \frac{d^l f(x)}{dx^l}|_{x=0}, l = 0, 1, 2$$

2) Continuity Constraints:

$$(h_j)^{1-l} c_j^{n,l} = (h_{j+1})^{1-l} c_{j+1}^{0,l}, l = 0, 1, 2, j = 0, 1, \dots, m-1.$$

3) Safety Constraints:

$$\underline{p}_j^0 + h_j \underline{p}_j^1 M_{i,1} \leq h_j c_j^{i,0} \leq \bar{p}_j^0 + h_j \bar{p}_j^1 M_{i,1}$$

4) Physical Constraints:

$$\underline{\beta}_j^l \leq (h_j)^{1-l} c_j^{i,l} \leq \bar{\beta}_j^l$$

5) Kinematic Speed Constraints:

$$c_j^{i,1} \leq min\{\bar{\beta}_j^1, \sqrt{a_{lat}^{des}/|k|_{r,max}}\}$$

6) Kinematic Path Constraints:

$$h_j c_j^{i,0} \leq max\{\frac{1}{k_r} - \frac{l_{wb}}{tan(\delta_{max})}, l_{fu}\} \text{ if } k_r > 0$$

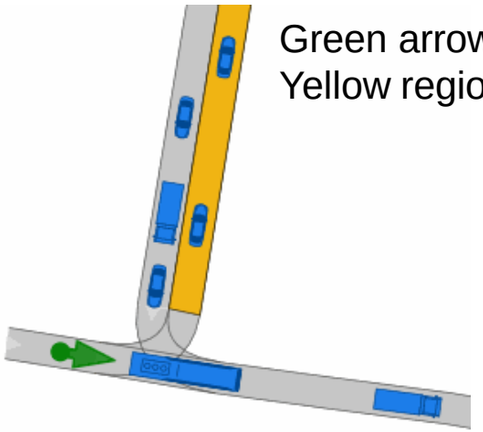
$$h_j c_j^{i,0} \geq min\{\frac{1}{k_r} + \frac{l_{wb}}{tan(\delta_{max})}, l_{fl}\} \text{ if } k_r < 0$$

Validation Scenario 1: Urban T-intersection

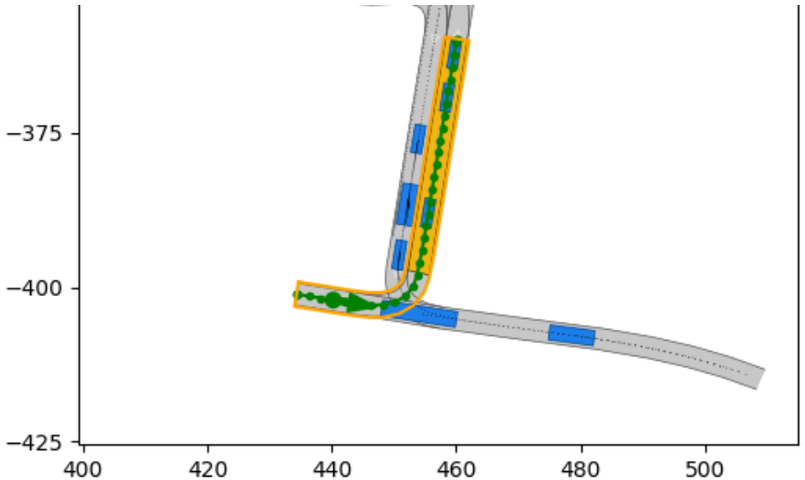
A nominal planner (search-based planner) provides a reference trajectory.

However, the intention of other vehicles changes, the reference trajectory must be adapted.

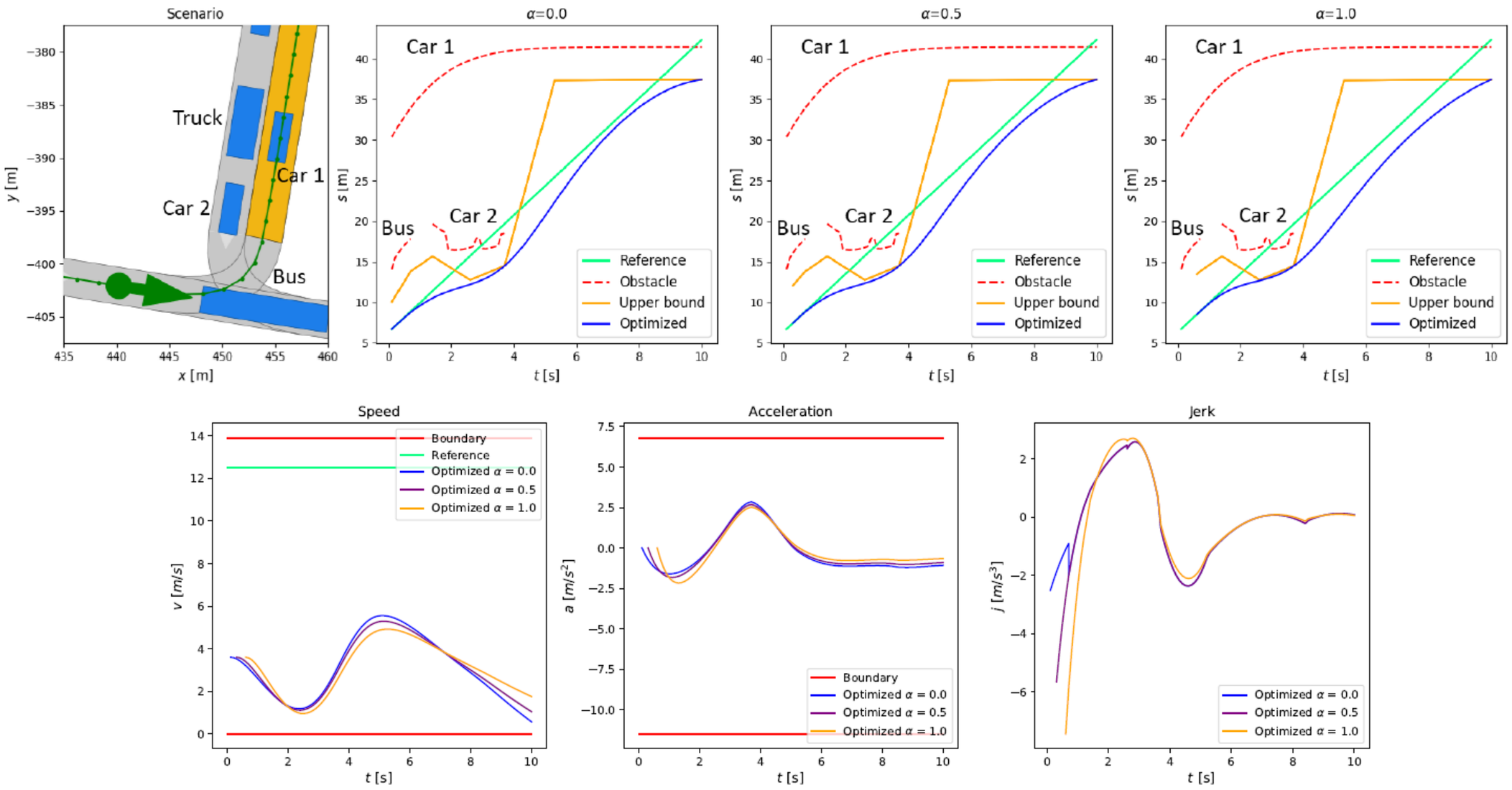
The obstacle vertices are projected onto S-T domain.



Green arrow: initial state
Yellow region: goal region



Speed optimization results

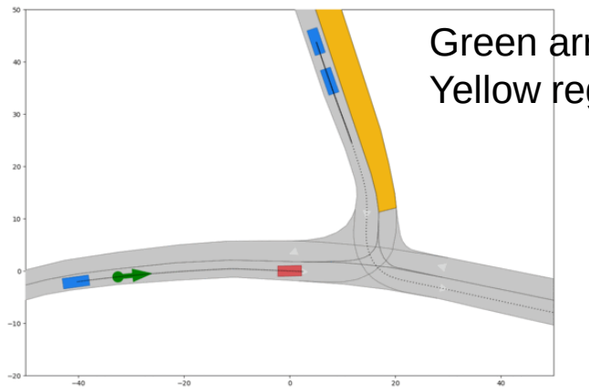


Validation Scenario 2: Blocked T-intersection

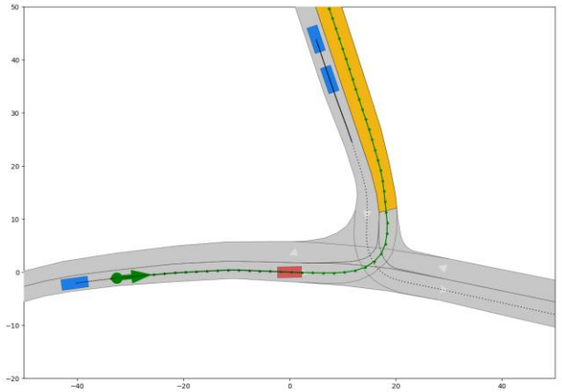
A nominal planner (search-based planner) provides a reference trajectory

A static obstacle suddenly blocks the road.

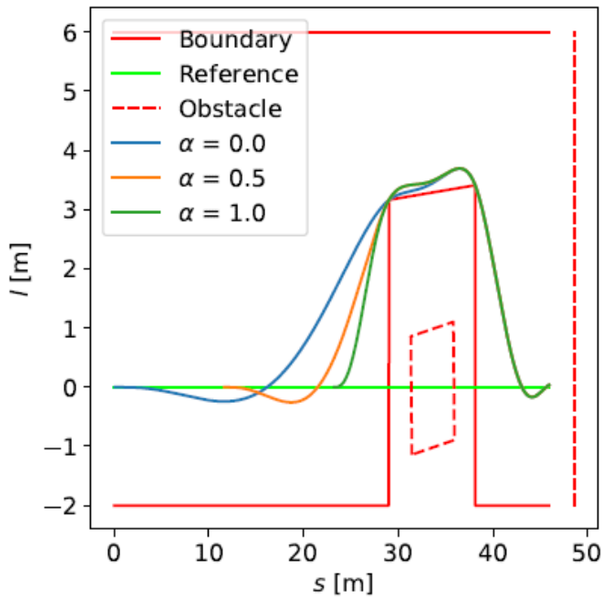
Occupancy of other vehicles in planning horizon is projected into S-L domain



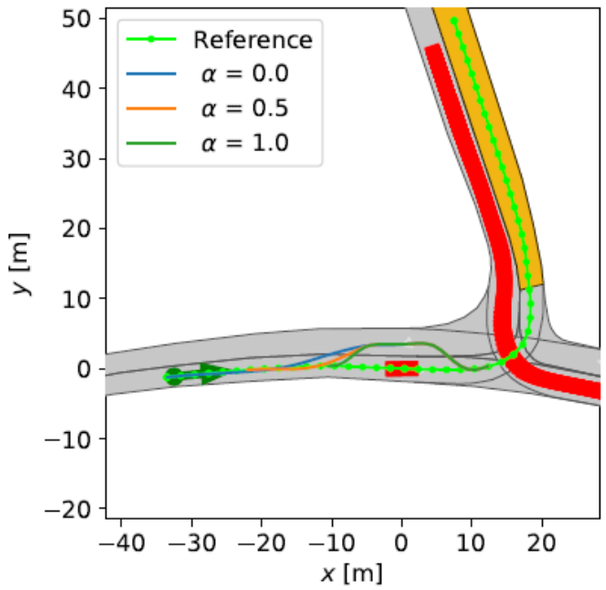
Green arrow: initial state
Yellow region: goal region



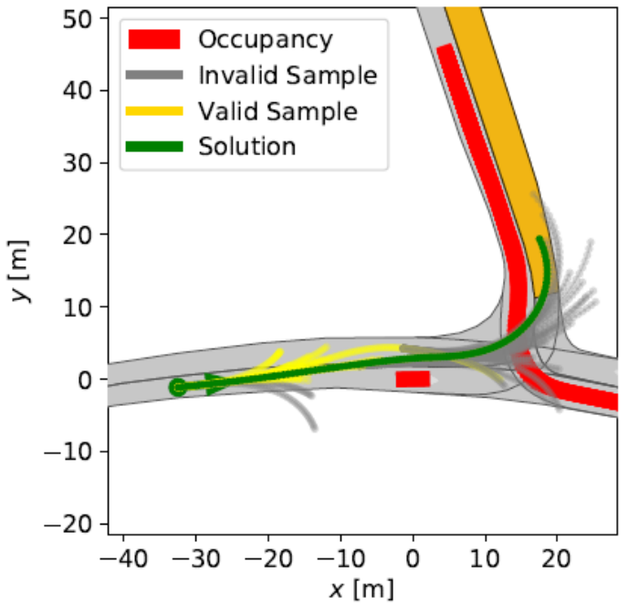
Path optimization results



(a) Path repairing in L-S domain



(b) Path repairing in X-Y domain



(c) Sampling of CL-RRT in X-Y domain

TABLE I: Comparison of curvature.

	α=0	α=0.5	α=1	CL-RRT	Maximal
Maximal Curvature	0.46	0.46	0.45	0.10	0.54
Average Curvature	0.06	0.08	0.13	0.03	0.54

Computation time comparison

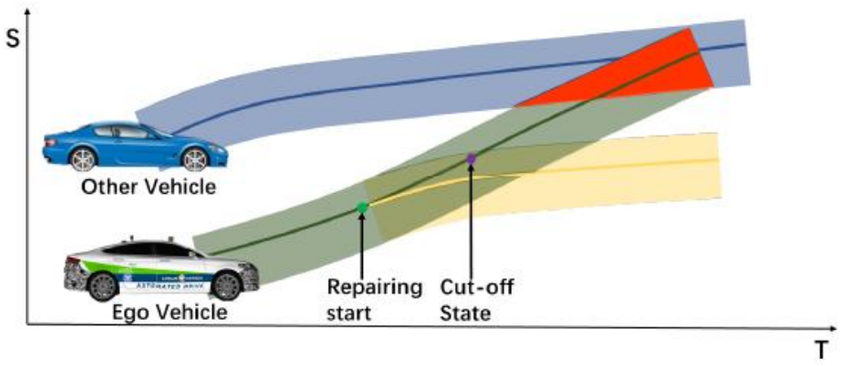
TABLE II: Comparison of computation time. We run 100 iterations for each algorithm. Speed and path refer to speed repairing and path repairing, including the computation time for generating trapezoidal corridors and establishing and solving the optimization problem. The number before and after ± are the average and standard deviation respectively.

Scenario	Cut-off State		$\alpha = 0$		$\alpha = 0.5$		$\alpha = 1$		CL-RRT
	TTR	DTR	Speed	Path	Speed	Path	Speed	Path	
(1)	9.8±0.4ms	-	9.2±2.7ms	-	9.8±1.0ms	-	13.8±1.2ms	-	TIMEOUT
(2)	11.4±1.8ms	1.8±0.0ms	18.7±1.8ms	6.1±0.2ms	18.8±1.8ms	6.0±0.3ms	18.8±1.4ms	6.2±0.7ms	TIMEOUT

Python implementation
Optimization Solver: OSQP
CPU: Intel(R) Xeon(R) W-2123 CPU @ 3.60GHz

Key takeaways

- **Efficient and real-time trajectory repairing framework**
- **Bézier curve optimization with trapezoidal corridors and speed/path kinematic constraints**
- **A handy parameter (α) for behavior tunning.**



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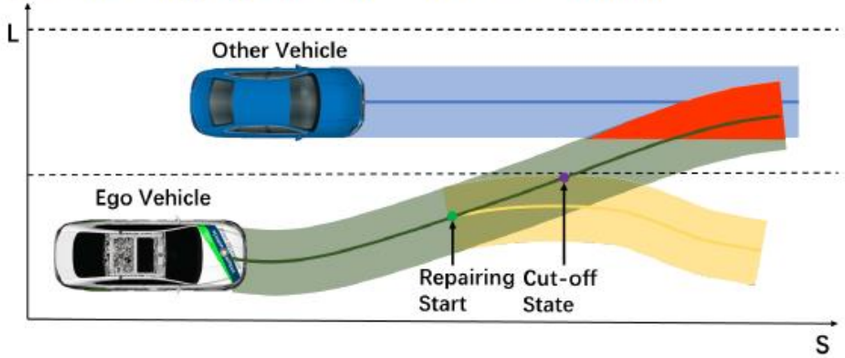

(b) Path Repairing in L-S domain. The motion of the ego vehicle and the other vehicle is projected into the L-S domain in curvilinear coordinates.

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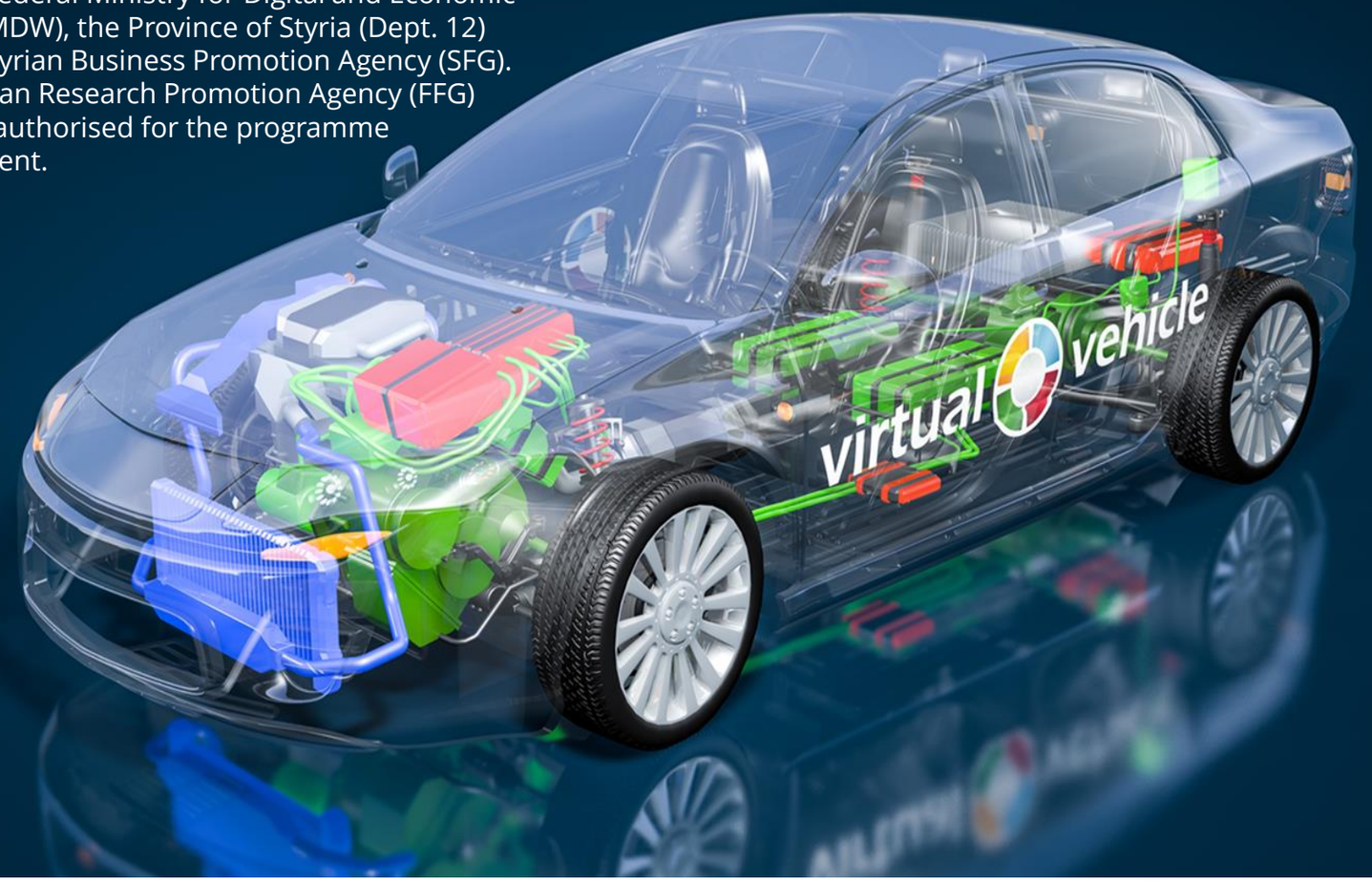

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